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20MCA14

First Semester MCA Degree Examination, Jan./Feb. 2021
Mathematical Foundation for Computer Applications

Time: 3 hrs.

Max. Marks: 100

Note: *1. Answer FIVE full questions, choosing ONE full question from each module.*
2. Use of Statistical table is permitted.

Module-1

1 a. Define a set, empty set and a singleton set with example for each. **(04 Marks)**
   b. Define union and intersection of two sets with example. **(04 Marks)**
   c. Find the eigen values and eigen vectors of the matrix.

$$\begin{pmatrix} 2 & 1 & -2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

   (12 Marks)

OR

2 a. What is the cardinality of a set? Find the cardinality of the sets A and B where $A = \{2, 3, 4, 5, 6, 7, 8, 9, 9\}$ and $B = \{a, e, i, o, u\}$. **(04 Marks)**
   b. A total of 1232 students have taken a course in Java, 879 in C and 114 have taken a course in C++. Further 103 have taken courses in both Java and C, 23 have taken courses in both. Java and C++ and 14 have taken courses in both C and C++. If 2092 students have taken atleast one of Java, C and C++, how many students have taken a course in all the three subjects. **(08 Marks)**
   c. For any three sets A, B, C prove that i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$ ii) $\overline{A \cap B} = \overline{A} \cup \overline{B}$ **(06 Marks)**
   d. State and explain Pigeon hole principle. **(02 Marks)**

Module-2

3 a. What is a Proposition? Let p and q be the propositions "swimming in the new jersy seashore is allowed and sharks have been near the sea shore". Express each of the following compound propositions as an English sentence.
      i) $p \to \sim g$ ii) $\sim p \to \sim q$ iii) $p \leftrightarrow q$ **(06 Marks)**
   b. Write the contra positive, the converse and the inverse of the conditional statement "If the home team wins, then it is raining". **(06 Marks)**
   c. Show that the compound proposition $[(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)]$ is logically equivalent to $[(p \to q) \wedge (q \to r) \wedge (r \to p)]$. **(08 Marks)**

OR

4 a. Show that the following argument is valid. If Today is Tuesday, I have a test in Mathematics (or) Economics. If my Economics professor is sick, I will not have a test in Economics. Today is Tuesday and my Economics professor is sick. Therefore I have a test in Mathematics. **(08 Marks)**
   b. Give the proof of the following statement, "If n is an odd integer, then n^2 is odd using direct and indirect proof method". **(07 Marks)**
   c. What is the truth value of $\forall x(x^2 >= x)$,
      i)   If the domain consists of all real numbers.
      ii)   If the domain consists of all integers. **(05 Marks)**

Module-3

5 a. Let A = {1, 2, 3, 4}, let R = {(1, 3), (1, 1), (3, 1), (1, 2), (3, 3), (4, 4)} be a relation on A. Determine whether R is reflexive, symmetric, anti-symmetric (or) transitive. **(08 Marks)**

   b. Give the directed graph of the relation R = {(1, 1), (1, 3), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (4, 1)} on the set {1, 2, 3, 4} **(04 Marks)**

   c. Let R_1 and R_2 be the relations represented by the matrices

$$M_{R_1} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \text{ and } M_{R_2} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

   Determine:
   i) $R_1 \cup R_2$ ii) $R_1 \cap R_2$ iii) R_1 iv) R_2 **(08 Marks)**

OR

6 a. Discuss briefly on partitions and equivalence classes. **(08 Marks)**

   b. Draw the Hasse diagram representing the partial ordering {(a, b)/a divides b} on {1, 2, 3, 4, 6, 8, 12}. **(12 Marks)**

Module-4

7 a. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6
P(x)	k	3k	5k	7k	9k	11k	13k

   i)   Find k
   ii) Evaluate $P(X < 4)$, $P(X \geq 5)$, $P(3 < X \leq 6)$.
   iii) Find the minimum value of K so that $P(X \leq 2) > 0.3$. **(11 Marks)**

   b. The probability that a pen manufactured by a company will be defective is 1/10. If 12 such pens are manufactured find the probability that i) exactly two will be defective   ii) atleast two will be defective   iii) None will be defective. **(09 Marks)**

OR

8 a. For the probability density function f(x), where

$$f(x) = \begin{cases} x^2/3, & -1 < x < 2 \\ 0, & \text{else where} \end{cases}$$

   find F(x) and use it to evaluate $P(0 < X \leq 1)$. **(06 Marks)**

   b. The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted the diseases. What is the probability that
   i)   Atleast 10 survive
   ii) From 3 to 8 survive
   iii) Exactly 5 survive. **(09 Marks)**

   c. Given a standard normal distribution find the value of K such that
   i) $P(z > K) = 0.3015$
   ii) $P(K < z < -0.18) = 0.4197$ **(05 Marks)**

Module-5

9 a. Define the following with suitable examples:
   i)   Simple graph
   ii) Complete graph
   iii) Bipartite graph
   iv) Complete bipartite graph. **(06 Marks)**

b. Check whether the following 2 graphs are Isomorphic with each other. (06 Marks)
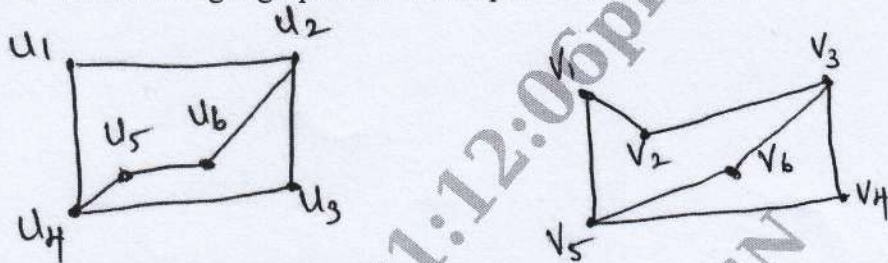
c. Explain the Konigsberg bridge problem. (08 Marks)

OR

10 a. Define the terms:
   i) Hamilton path
   ii) Eulers path
   iii) Planar graphs
   iv) Subgraph of a graph with suitable example for each. (06 Marks)

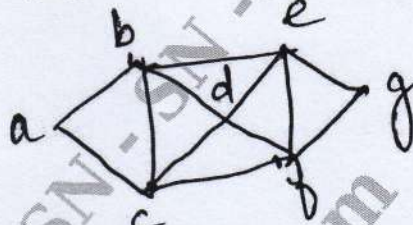b. Give the graph colouring of the graph shown in Fig.Q.10(b).

Fig.Q.10(b)

(04 Marks)

c. Use Dijkstra's algorithm to find the length of a shortest path between the vertices a and z in the graph given below, shown in Fig.Q.10(c).
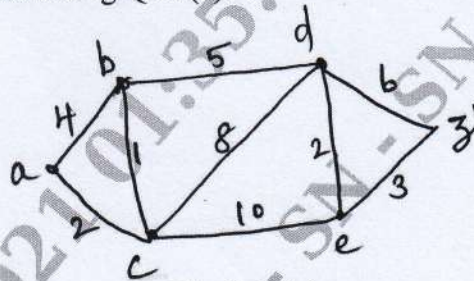

Fig.Q.10(c)

(10 Marks)

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